How we teach calculations in Maths
A Parent's Guide

Belmont Maths Department

2011

Contents

Introduction

Many parents, keen to support their children, have asked about the calculation methods we teach at Belmont. We have produced this booklet which we hope will explain the methods your children use and why.

Overview:

The ability to calculate mentally forms the basis of all calculations and therefore, up until the end of Year 3, the focus in class is on teaching these mental methods.
Throughout KS1 and the beginning of KS2, pupils build up their counting strategies and develop a secure understanding of place value and number facts. They use a range of practical activities and are encouraged to jot things down informally or use resources (such as number lines or 100 squares) to help them.
As the pupils become more confident, their mental strategies will increase in speed and accuracy and they may no longer need resources to help them.

It is only once the children are confident in these mental strategies that they begin to look at more formal written methods. As they move through KS2, the children are taught a number of techniques, initially expanding on their mental strategies before progressing towards an efficient and more compact written method.

Our ultimate aim is to ensure that all children have a reliable written method with which they feel confident using when they are unable to carry out the calculation mentally.
It is important to understand that our brains don't all function in the same way - different children (and adults) will prefer to use different methods. By teaching a wide range of calculation strategies, children are given the opportunity to choose one that they understand and feel comfortable with, which ultimately increases their confidence and accuracy.

Please encourage your child to use the method with which they feel most comfortable and which they can use most reliably.

It is useful to ask your child to explain the method they are using as it helps them to a clearer understanding.

Maths at Belmont

The overall aim within our maths teaching at Belmont is that by the time children begin Year 7 they:

- have a secure knowledge of number facts and a good understanding of the four operations of addition, subtraction, multiplication and division;
- are able to use this knowledge to carry out calculations mentally;
- make use of diagrams and informal notes (jottings) to help record steps in mental methods;
- have a reliable written method, (as compact and efficient as possible) for each of the four operations (addition, subtraction, multiplication and division);
- use a calculator effectively, decide if the numbers displayed make sense and interpret them accurately.

Oral and Mental Maths

As mentioned above, oral and mental work is essential in laying the foundations for written calculations. Oral and mental skills are practised, rehearsed and built on in all year groups throughout Belmont.

Secure mental calculation requires the ability to:

- recall key number facts instantly – e.g. number bonds to 10, 100 etc and multiplication facts up to 10 ×10
- use knowledge of place value to extend known number facts – e.g. using the fact that 7 x 6 = 42 to calculate 70 x 6 = 420;
- use strategies such as realising that addition or multiplication can be done in any order, or partitioning numbers into hundreds, tens and units.

By becoming confident in a selection of different methods, pupils are encouraged to tackle problems by finding the most reliable, efficient way of approaching a calculation. For example, if the question requires you to find 800 – 198, it may be best to count on from 198 to 200, and then on to 800 rather than count backwards from 800.

Addition

In order to add successfully, children must be able to do the following:

- recall all addition pairs to 9 + 9 and which pairs of numbers add to make 10;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition (or split) two-digit and three-digit numbers in different ways (see Step 1 below)

Some mental strategies:

- Add tens first (e.g. 42 + 35 = 42 + 30 + 5)
- Using near doubles (e.g. 45 + 46 = 45 + 45 + 1)
- Using "near numbers" (e.g. 37 + 199 ⇨ 37 + 200 – 1 = 237 – 1 = 236)
- Finding units digits that make 10 first (e.g. 46 + 19 + 24 ⇨ 70 + 19 = 89)

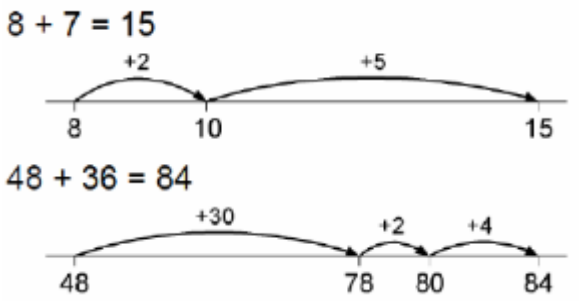
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads, they learn to use an efficient written method.

The following shows the steps in building up to using an efficient written method for addition of whole numbers.

Step 1: Number lines

An empty number line is just a line drawn by the child, without numbers marked on it. The child then writes on only the numbers needed to help with the calculation.

Children may need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.



The steps often *bridge* through a multiple of 10. (This means splitting up the number in such a way that the pupil can reach a multiple of 10.)

Step 2: Partitioning

The children are now shown how to record their mental partitioning method:

47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123

The children are then encouraged to record this in columns:

$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +\,76 & & 70 + 6 \\ \hline & & 110 + 13 \end{array} = 123$$

This method emphasises the value of each digit and ensures children understand what they are doing and why.

Step 3: Expanded column method

The children can now move towards a more formal method.
This layout still uses the addition of tens to tens and units to units separately; initially, the children are allowed to do this in any order they wish, but as they become more confident they are asked to always add the units first.

$$\begin{array}{r} 47 \\ +\,76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array} \quad \text{or} \quad \begin{array}{r} 47 \\ +\,76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$

We will always emphasise the addition of the tens as 40+70 not 4+7.

Step 4: Column method

This is the final, most compact method.
However, children will only move on to it once they have secured the expanded method.

$$\begin{array}{r} 47 \\ +\,76 \\ \hline 123 \\ 1\,1 \end{array} \qquad \begin{array}{r} 258 \\ +\,87 \\ \hline 345 \\ 1\,1 \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ 1\,1 \end{array}$$

This method reduces recording even further. Carry digits are recorded below the line using the words ‘carry ten’ or ‘carry one hundred’ and not ‘carry one’. Once children have mastered this method, they will find it quick and reliable. They will be able to use it for larger whole numbers as well as decimals.

subtraction

In order to add successfully, children must be able to do the following:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70 = 90 because 16 – 7 = 9);
- partition or split numbers into hundreds, tens and units in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Some mental strategies:

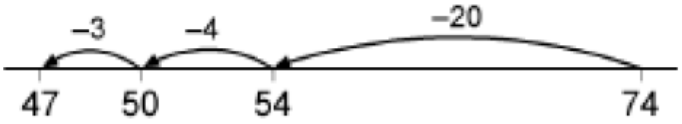
- Subtract tens then units (e.g. 67 – 35 = 67 – 30 – 5 =32)
- Use Near doubles (e.g. 54 – 28 = 54 – 27 – 1 = 26)
- Use a "Nearly number" (e.g. 60 – 19 = 60 – 20 + 1 = 41)
- Count up from the smaller number (e.g. 2005 – 1996 ⇨ Count forward from 1996 to 2000 (4) then to 2005 (5) ⇨ 5 + 4 = 9)

The aim, as for addition, is that children use mental methods when appropriate, but as they move through the school, they begin to use written methods for calculations that they cannot do in their heads. The following shows the steps in building up to using an efficient method for subtraction of two-digit and three digit whole numbers.

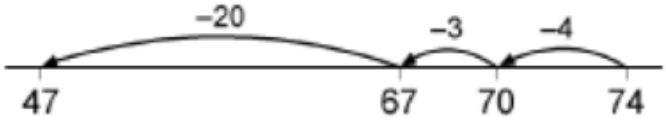
Step 1: The empty number line

Again, the children start by informally recording their mental methods onto a number line:

e.g. 74 – 27 = 47 worked by counting back:



The steps may be recorded in a different order:

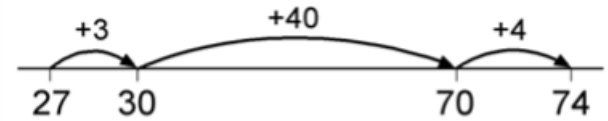


The children can record each step of their calculation and, with time and practice, will need to record less information.

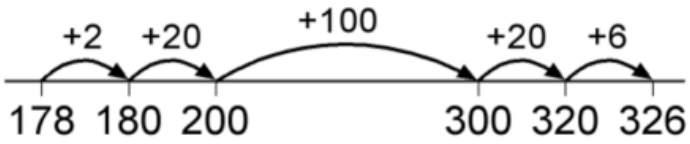
Step 2: The counting up method

The children are taught the concept of subtraction as the difference between two numbers. This difference can then be calculated by counting up from the smaller number to the bigger one:

e.g. 74 – 27 = 47

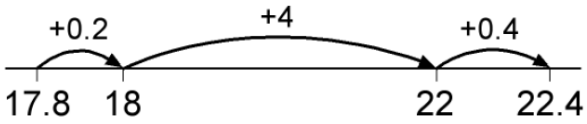


326 – 178



It can also be used with decimal numbers:

22.4 – 17.8



The number of steps the children take will be reduced as they gain experience.

Step 3: Partitioning

With this method, the children are encouraged to partition the number that is being taken away in order break the calculation down into smaller steps:

$$74 - 27 = 74 - 20 - 7$$
$$74 - 20 = 54$$
$$54 - 7 = 47$$

In this case, the 27 has been partitioned to 20 and 7 so that the tens can be taken away first and then the units.

Step 4: Expanded column method

This method parallels the column addition method.
There are simple calculations:
e.g. 567 – 123

$$\begin{array}{rrrrrrr} = & 500 & + & 60 & + & 7 & \\ - & 100 & + & 20 & + & 3 & \\ \hline & 400 & + & 40 & + & 4 & = 444 \end{array}$$

Or ones that require the children to repartition the numbers:
74 – 27

$$\begin{array}{r} 70 + 4 \\ - \ 20 + 7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 60} \quad {\scriptstyle 14} \\ \cancel{70} + \cancel{4} \\ - \ 20 + 7 \\ \hline 40 + 7 \end{array}$$

As 7 cannot be taken away from 4, a ten is borrowed to make it 14 instead.

Step 5: Column method

This compact method requires mental partitioning of the numbers rather than written:

$$\begin{array}{r} 500 + 60 + 3 \\ -\ 200 + 40 + 1 \\ \hline 300 + 20 + 2 \end{array}$$

becomes

$$\begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$$

The children are encouraged to say 60 – 40 instead of 6 - 4

Often, adjustments will need to be made:

$$\begin{array}{r} {\scriptstyle 400}\quad {\scriptstyle 160} \\ \cancel{500} + \cancel{60} + 3 \\ 200 + 70 + 1 \\ \hline 200 + 90 + 2 \end{array}$$

$$\begin{array}{r} {\scriptstyle 4}\ {\scriptstyle 16} \\ \cancel{5}\ \cancel{6}\ 3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

In this case, 70 cannot be subtracted from 60 so one hundred is borrowed to make it 160 (or 16 tens)

Multiplication

To be able to multiply successfully, children need to be able to:

- recall all times tables to 10 × 10;
- partition, or split, numbers into multiples of one hundred, ten and one;
- identify and apply rules for multiplying by 10, 100 and 1000.
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7;
- add whole numbers using a written method.

Mental strategies:

- Multiplying by 10, 100, 1000 etc (see P12 for more information)
- Using doubling (e.g. 37 x 4 ⇨ double 37 (74) then double again (148))
- Using halving (e.g. 84 x 5 = Half of (87 x 10) = Half of 870 = 420
  e.g. 54 x 25 ⇨ ((54 x 100) then halve twice = 1350)
- Using doubling and halving (e.g. 35 x 14 = 70 x 7 = 490 (double the odd number, halve the even number)
- Using rounding and adjusting (e.g. 26 x 99 = (26 x 100) - 26 = 2574
  e.g. £1.99 x 3 = (£2 x 3) – 3p = £5.97

As with the other operations, the aim is that children use mental methods when appropriate, but they use an efficient written method accurately for calculations that they cannot do in their heads. The following show the stages in building up to using an efficient method for multiplication of two-digit and three-digit whole numbers.

Step 1: Partitioning

By partitioning the number, children can multiply each part separately.

48 x 6 becomes:

40 x 6 = 240 (using knowledge of 4 x 6)
8 x 6 = 48
= 288

Step 2: Grid Method

This is an alternative way of organising the partitioning method.

x	6
40	240
8	48
	288

It is a very useful method for more complicated calculations too:

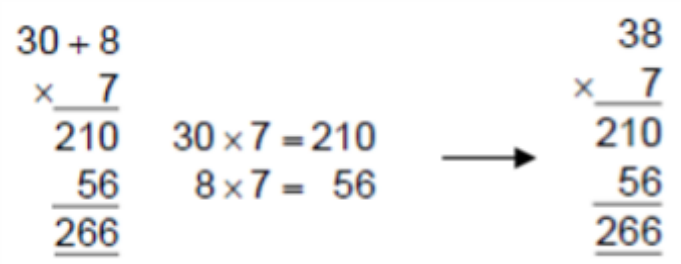
36 x 24

x	30	6	
20	600	120	720
4	120	24	144
			864

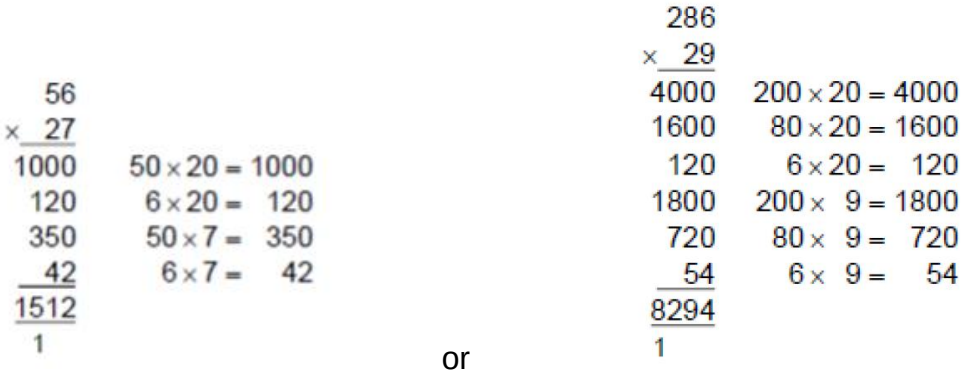
Many children prefer to use this method over any other. It is a very useful method and can also be applied to decimal calculations.

Step 3: Expanded column multiplication

The next step is to represent the method of recording in a column format.

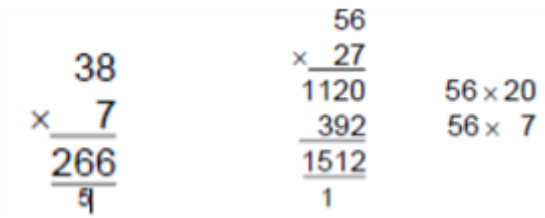


This method can be used for longer, more complicated calculations as well:



Step 4: Compact method of Multiplication

Here, recording is further reduced, with carry digits written below the line:



If, after practice, the children cannot use this method without making errors, they should return to the expanded method.

A note about place value

Place value refers to the fact that digits have different values depending on their place in a number.
For example, the 3 in 37 means 30 because it is in the tens column, whereas the 3 in 356 means 3 hundred.

Pupils will be taught to use place value charts to help them multiply and divide numbers by 10, 100, 1000 etc. They will see that as you move from right to left in the chart, the digits are worth 10 times more each time.

Unlike in "the old days", the decimal point is FIXED and therefore the digits are moved to the left when a number is multiplied and will move to the right when the number is divided by 10, 100, 1000 etc.

e.g. 6.3 x 10 = 63

TH	H	T	U	.	Tenths	Hundredths
			6	.	3	
		6	3			

Try to avoid saying to your child "just add a zero" when you multiply by 10 as this does not work with decimals.

e.g. 4.5 x 10 ≠ 4.50 as this is the *same* number

Division

To be able to divide successfully, children need to be able to do the following:

• partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
• recall multiplication and division facts to 10 × 10, recognise multiples of one digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
• understand division as repeated subtraction, and perform subtraction reliably;
• estimate how many times one number divides into another – for example, how many sixes there are in 47;
• multiply a two-digit number by a single-digit number mentally.

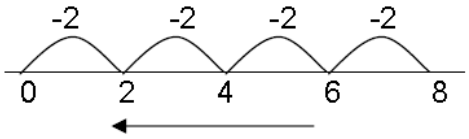
Some mental strategies:

- Dividing by 10, 100, 1000 etc (See P12 for further information on this)
- Using halving (e.g. 54 ÷ 4 ⇨ halve 54 (27) halve again (13.5))
- Using factors (e.g. 620 ÷ 20 = 31 (divide by 2 then divide by ten using factors of 20)

As with the other operations the aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. These notes show the stages in building up to using an efficient method for division of two-digit and three-digit whole numbers.

Step 1: (HTU ÷U) Chunking method

The principle behind the following division method is that division is repeated subtraction. For example, when doing 8 ÷ 2, younger children will be encouraged to keep taking away 2 until they no longer can.



As the numbers get bigger, children will begin to see this as a very time consuming process and will start to take away bigger chunks of the number instead to make it quicker.

196 ÷ 6

6)196		
− 60	6×10	
136		
− 60	6×10	
76		
− 60	6×10	
16		
− 12	6× 2	
4	32	
Answer:	32 R 4	

Instead of taking away 6 each time, the child is taking ten lots of 6 away in one go.

Further still, children will begin to realise that the fewer subtractions they have to do, the quicker and more efficient the strategy is. They will therefore learn to work with even bigger chunks:

$$
\begin{array}{rl}
6\overline{)196} & \\
-\ 180 & 6\times 30 \\
\quad 16 & \\
-\ \underline{12} & 6\times \underline{2} \\
\quad 4 & \quad 32 \\
\text{Answer:} & \quad 32\,\text{R}\,4
\end{array}
$$

Step 2: Short division

'Short' division of HTU ÷ U can be introduced as an alternative, more compact recording.

For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split 291 into 270 + 21. Each part is then divided by 3.

291 ÷ 3 = (270 + 21) ÷ 3
= (270 ÷ 3) + (21 ÷ 3)
= 90 + 7
= 97

The short division method is recorded like this:

$$3\overline{)290+1} = 3\overline{)\begin{array}{r}90+7\\270+21\end{array}}$$

This is then shortened to:

$$3\overline{)\begin{array}{r}97\\2\,9^{2}1\end{array}}$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 ones.

We begin by asking 'How many threes in 290?' (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7.

Step 3: Long division (HTU ÷TU) Expanded method

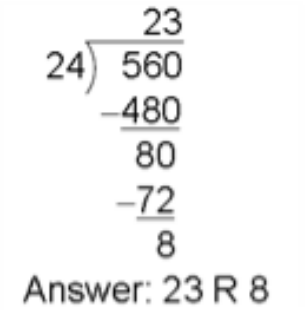
This method, in essence, is the same as the chunking method.

$$
\begin{array}{rl}
\quad 23 & \\
24\overline{)\ 560} & \\
-480 & 24\times 20 \\
\quad 80 & \\
-72 & 24\times 3 \\
\quad 8 & \\
\text{Answer: } 23\text{ R }8 &
\end{array}
$$

Children find the largest multiple of 24 which is also a multiple of 10 and is smaller than the number being divided.
e.g. 24 x 20 = 480
24 x 30 = 720 (too big)
480 will be the first chunk.
Appropriate chunks are then found for the rest of the calculation.

Step 4: Long division (HTU ÷TU)

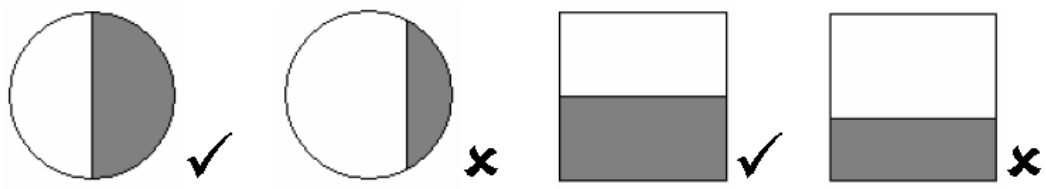
In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below:



Answer: 23 R 8

Fractions

Work with fractions will start with opportunities for the pupils to divide objects into halves. (NB It is important to emphasise the point that the two pieces must be of the same size for them to be classed as halves.)



The same rule applies to other fractions e.g. with quarters all 4 pieces must be the same size.

Writing Fractions

Written fractions can appear complicated to children. One way of explaining them is to compare the line in the fraction to the line in the divide sign (÷). It is explained to pupils that the line is the "cutting" line and the number below the line (the denominator) is how many pieces you cut the shape into.

The number above the line (the numerator) shows how many pieces we have e.g.

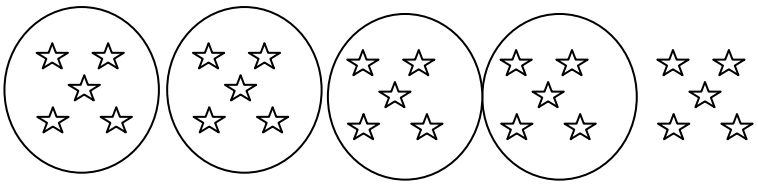
½ means we have 1 of 2 pieces
¼ means we have 1 of 4 pieces
¾ means we have 3 of 4 pieces etc

Fractions of Numbers

Pupils will being by finding simple fractions of amounts (such as ½ of 12 or ⅓ of 18) by dividing objects (such as counters) up into the required number of groups.

This obviously links in very closely with times tables' knowledge.

This then leads on to the pupils finding other fractions such as $^{4}/_{5}$ of 25.

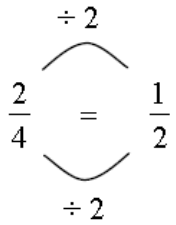


Equivalent Fractions

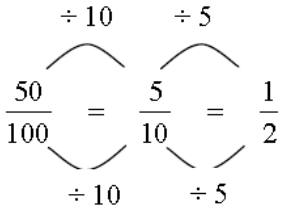
e.g. ½ = $^{2}/_{4}$ = $^{3}/_{6}$ = $^{10}/_{20}$ = ...

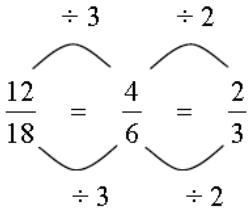
In the early stages pupils are given lots of opportunities to colour in different shapes and to cut up shapes so that the various fractions can be compared. As they progress with their fractions work they will see that some fractions represent the same amount, for example, if you eat two quarters of a pizza, you have eaten one half of the pizza.

Later they are shown how to 'reduce the fraction down to its lowest terms' by dividing both parts of the fractions (the numerator and the denominator) by the same number.



Sometimes it is easier to reduce the fraction down in stages e.g.





Decimals and Percentages

The link between fractions, decimals and percentages is an extremely important one. Pupils will be required to know how to convert between all three.

By the time they are in Years 7 and 8, pupils will be using these facts to answer questions such as:

Find 78% of £300 without a calculator:

50% = ½ of £300 = £150
25% = ¼ of £300 = £75
1% = $^{1}/_{100}$ of £300 = £3
So 3% = 3 x £3 = £9

Therefore 78% = £150+£75+£9 = £234

Using Calculators

Children begin to use calculators from around Year 5 and onwards. They will:

- Use calculators to check their work;
- Explore patterns in number using a calculator;
- Learn when the calculator is the most appropriate method to perform a calculation.

It must be noted that one problem with using calculators is that children have a tendency to trust the calculator even beyond their common sense. Pupils are encouraged to always check their answers "make sense" and not to necessarily take the answer on the calculator as read.

There are a couple of other areas which children often find hard when using the calculator:

Rounding Up and Down

169 children are on a camping holiday.
Each tent holds 9 children.
How many tents are needed?
Using a calculator gives the following answer.
169 ÷ 9 = 18.777777
Since 18.777777 tents do not make sense, you need to consider whether it is 18 or 19 tents that are required. If they only took 18 tents, some children would have nowhere to sleep, so in this case it is necessary to *round up* the answer to 19 tents.

A farmer collects 65 eggs from her chickens.
How many boxes of 6 eggs can she fill?
The calculator gives 65 ÷ 6 = 10.833333
In this case, you need to *round down*, because only 10 boxes can be filled. The eleventh box would only have 5 eggs in it.

Calculations Involving Money or Units of Measurement

£34 + 52p does not equal £86 or 86p.
The pupils are taught to change all the amounts into the *same* units.
i.e. the solution to the question would be:

In pounds (£)	34 + 0.52 = 34.52 which is then interpreted as £34.52
Or in pence	3400 + 52 = 3452 which is then calculated as £34.52

The same rule applies when working with metres and centimetres, or other mixed units – make sure all the values have been converted to the same unit.
Also, when adding £3.45 + £2.15, the calculator will give the answer 5.6
However it is wrong to give the answer as £5.6 – the correct answer is £5.60

Glossary

Factor	a number that will divide into another number without leaving a remainder
Partitioning	breaking down a number into its place value parts eg 34 into 3 tens and 4 units
Number line	horizontal (or vertical) representation of the number sequence
Chunking	A form of division based on the fact that division is repeated subtraction
Place value	Recognizing that the value of a digit is determined by its *place* in a number
Bridging	Working out what should be added or taken away to bring you to a 10 or multiple of 10. (Helpful technique in adding and subtracting)
Decomposition	To partition numbers in different ways to allow subtraction to take place when the subtracting number is larger than the starting number e.g. 700+80+4 is the same as 700+70+14. (what you may know as borrowing)
Remainder	The number left when you cannot equally divide (share)
Inverse	The 'opposite' calculation or working the calculation backwards
Difference	The result of subtracting one number from another
Array	an arrangement of a set of numbers or objects in rows and columns

websites

Some useful websites to use at home:

www.mymaths.co.uk
username: bmhps
password: angle

www.woodlands-junior.kent.sch.uk/maths

http://www.mathszone.co.uk/

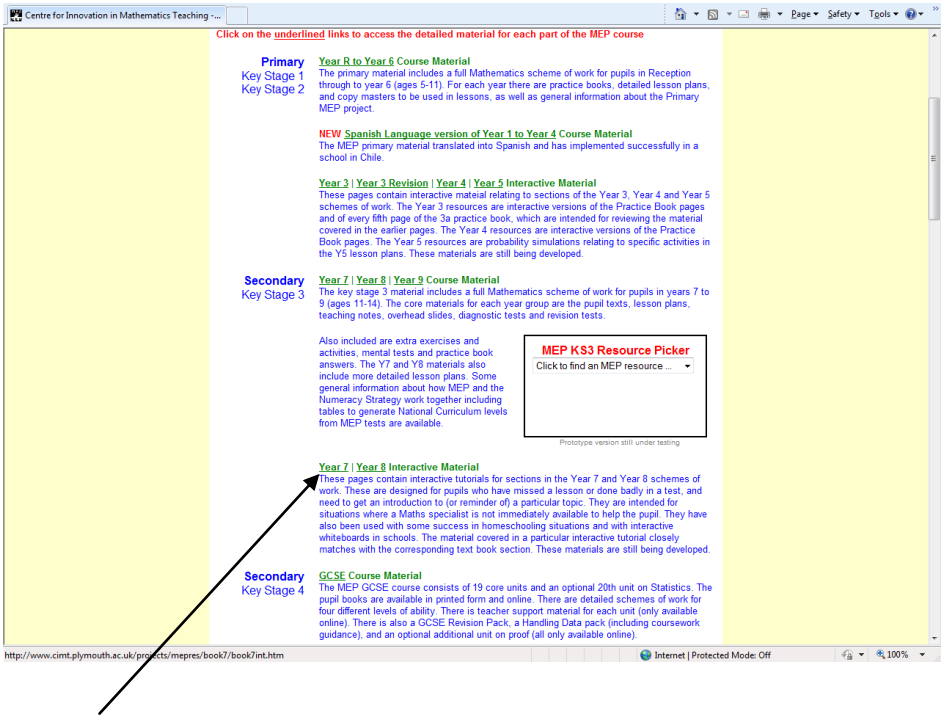
www.echalk.co.uk
login: Belmont
password: echalk

www.topicbox.co.uk/Maths/

www.bbc.co.uk/schools/ks2bitesize/maths/

www.mathsisfun.com

http://www.cimt.plymouth.ac.uk/projects/mep/default.htm
(more useful for Y7 and Y8 pupils)



This takes you to Y7 or Y7 interactive tutorials for different topics.